Problem 1.18

Elevator and falling marble
At $t = 0$, an elevator departs from the ground with uniform speed. At time T_1 a child drops a marble through the floor. The marble falls with uniform acceleration $g = 9.8\ \text{m/s}^2$, and hits the ground T_2 seconds later. Find the height of the elevator at time T_1.

Solution

Since the elevator moves at uniform speed v, it has no acceleration. The elevator floor's position from the ground after t seconds is given by

$$y = vt,$$

so after T_1 seconds the elevator floor is at a height of vT_1 meters. This is the initial position of the marble when the child releases it. Its initial velocity is the velocity of the elevator. Apply the kinematic formula,

$$y = y_0 + v_{0y}t + \frac{1}{2}at^2,$$

for the marble's motion from the elevator floor to the ground.

$$0 = vT_1 + vT_2 + \frac{1}{2}(-g)T_2^2$$

Solve this equation for v.

$$0 = v(T_1 + T_2) - \frac{g}{2}T_2^2$$

$$v(T_1 + T_2) = \frac{g}{2}T_2^2$$

$$v = \frac{g}{2}\frac{T_2^2}{T_1 + T_2}$$

Therefore, the height of the elevator at time T_1 is vT_1, or

$$\frac{gT_1}{2}\frac{T_2^2}{T_1 + T_2}.$$